

# Modelling of the 1D Convective Heat Exchange between Logs Subjected to Freezing and to Subsequent Defrosting and the Surrounding Environment

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**Abstract** – A 1D mathematical model for the computation of the temperature on the surface of cylindrical logs,  $t_{sr}$ , and the non-stationary temperature distribution along the radiuses of logs subjected to freezing and subsequent defrosting at convective exponentially changing boundary conditions has been suggested. The model includes mathematical descriptions of the thermal conductivity in radial direction,  $\lambda_r$ , the effective specific heat capacity,  $c_e$ , and the density,  $\rho$ , of the non-frozen and frozen wood, and also of the heat transfer coefficient between the surrounding air environment and the radial direction of horizontally situated logs,  $\alpha_r$ . With the help of the model, computations have been carried out for the determination of  $\alpha_r$ ,  $t_{sr}$ ,  $\lambda_{sr}$ , and 1D temperature distribution along the radiuses of beech logs with diameters of 0.24 m, initial temperature 20 °C, and moisture content 0.4 kg·kg<sup>-1</sup>, 0.8 kg·kg<sup>-1</sup>, and 1.2 kg·kg<sup>-1</sup>, during their freezing at -20 °C, and during subsequent thawing at 20 °C.

**heat transfer coefficient / surface temperature / temperature distribution / beech logs / radius**

**Kivonat** – Egydimenziós konvektív hővezetés modellezése fagyott és normál állapotú rönk és környezte között. Célunk egy 1D matematikai modell létrehozása volt, amely kiszámítja a hengeres farönk felületi hőmérsékletét,  $t_{sr}$ , és a rönk sugara menti hőmérséklet-eloszlást egy olyan hengeres farönkön, amelyet lefagyasztottak majd kiolvasztottak exponenciálisan változó hőátadási körülmények között. A modell magában foglalja a sugárirányú hővezetési tényező,  $\lambda_r$ , az effektív specifikus fajhő  $c_e$ , és a sűrűség  $\rho$  matematikai leírását nem-fagyott és fagyott állapotú faanyag esetében. Tartalmazza továbbá az  $\alpha_r$  radiális irányú hőátadási tényezőt a környező levegő és a vízszintesen fekvő rönk között. A modell segítségével számítások történtek az  $\alpha_r$ , a  $t_{sr}$ , és a  $\lambda_{sr}$ , valamint az 1D hőmérsékleteloszlás meghatározására 0,24 m átmérőjű bükk rönknél a sugár mentén a következő feltételek mellett: kezdeti hőmérséklet 20 °C, a nedvességtartalom értékei 0,4 kg·kg<sup>-1</sup>, 0,8 kg·kg<sup>-1</sup> és 1,2 kg·kg<sup>-1</sup>, a -20 °C-os fagyasztás során és az ezt követő felolvasztás folyamán 20°C-ig.

**hővezetési együttható / felületi hőmérséklet / hőmérsékleteloszlás / bükk rönk / sugár**

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## 1 INTRODUCTION

Logs prepared for veneer production are subjected to freezing and thawing in natural air conditions during the winter. The duration time and energy needed for the thermal treatment of frozen logs with the goal of plasticizing depends greatly on the level of freezing within the logs. (Sergovsky 1975, Shubin 1990, Trebula – Klement 2002, Videlov 2003, Deliiski 2004, 2009).

In the accessible specialized literature there are very few reports about the temperature distribution in frozen logs subjected to thawing (Steinhagen 1986, 1991, Steinhagen et al. 1987, Steinhagen – Lee 1988, Khattabi – Steinhagen 1992, 1993, 1995, Deliiski 2005, 2011, 2013b) and there is no information at all about the temperature distribution in logs during their natural or artificial freezing. That is why the modelling and the multi-parameter study of the processes of freezing and of the subsequent defrosting (thawing) of logs are of considerable scientific and practical interest.

The aim of the present work is to suggest a 1D mathematical model for the computation of the temperature on the surfaces of cylindrical logs,  $t_{st}$ , and the non-stationary temperature distribution along the radiuses of logs subjected to freezing and subsequent defrosting at convective exponentially changing boundary conditions. To achieve this goal, a base model of the heating and cooling processes of logs is used, one which has earlier been suggested and modified by the first co-author (Deliiski 2005, 2011, 2013b).

### Symbols:

$c$	= specific heat capacity ( $J \cdot kg^{-1} \cdot K^{-1}$ )
$D$	= diameter, m
exp	= exponent
$q$	= specific heat energy ( $kWh \cdot m^{-3}$ )
$r$	= radial coordinate: $0 \leq r \leq R$ , m;
$R$	= radius, m
$t$	= temperature ( $^{\circ}C$ ): $t = T - 273.15$
$T$	= temperature (K): $T = t + 273.15$
$u$	= moisture content ( $kg \cdot kg^{-1}$ ): $u = W/100$
$W$	= moisture content (%): $W = 100u$
$\alpha$	= heat transfer coefficient between the logs' surface and the air environment ( $W \cdot m^{-2} \cdot K^{-1}$ )
$\Delta$	= difference (for the temperature)
$\lambda$	= thermal conductivity ( $W \cdot m^{-1} \cdot K^{-1}$ )
$\rho$	= density ( $kg \cdot m^{-3}$ )
$\tau$	= time (s)
@	= at

### Subscripts and superscripts:

ad	= anatomical direction
b	= basic (for density, based on dry mass divided to green volume)
bw	= bound water
bwm	= maximum possible amount of bound water
c	= center (for the temperature or thermal conductivity on the logs' centers)
dfr	= defrosting (for the temperature of the defrosting medium)
e	= effective (for the specific heat capacity of the frozen and non-frozen wood)
exp	= exponent (for the time constant of the exponentially change in the air temperature)
fr	= freezing (for the temperature or for the duration of the freezing process or for the frozen state of the wood)

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fsp	=	fiber saturation point of the wood
fw	=	free water
m	=	medium (for the temperature of the freezing or defrosting air environment)
m0	=	initial (for the medium temperature at the end of the logs freezing or defrosting)
m1	=	end (for the medium temperature at the end of the logs freezing or defrosting)
nfr	=	non-frozen (for the state of the wood)
r	=	radial direction
sr	=	surface on the radial direction
0	=	initial (for the radial or time coordinates or for the average mass temperature of the logs at the beginning of the freezing process)
1	=	final (for the average temperature of the logs at the end of the defrosting process)
273.15	=	at 273.15 K, i.e. at 0 °C (for the wood thermal conductivity)
293.15	=	at 293.15 K, i.e. at 20 °C (for the fiber saturation point of the wood)

## 2 MATERIAL AND METHODS

### 2.1 Mechanism of the 1D heat distribution in the logs

The mechanism of the heat distribution in logs during their heating or cooling can be described by the equation of heat conduction (also known as the equation of Fourier-Kirchhoff). When the length of the logs exceeds their diameter by at least 3 or 4 times, then the heat transfer through the frontal sides of the logs can be ignored, because it does not influence the change in temperature of their cross sections which are equally distant from the frontal sides (Chudinov 1968). In such cases, the following 1D model can be used for the calculation of the change in  $T$  only along the radius of the central cross sections during freezing and defrosting of the logs (i.e. along the coordinate  $r$  of these sections) (Deliiski 2011, 2013b):

$$c_e \rho \frac{\partial T(r, \tau)}{\partial \tau} = \lambda_r \left( \frac{\partial^2 T(r, \tau)}{\partial r^2} + \frac{1}{r} \frac{\partial T(r, \tau)}{\partial r} \right) + \frac{\partial \lambda_r(r, \tau)}{\partial T} \left( \frac{\partial T(r, \tau)}{\partial r} \right)^2 \quad (1)$$

with an initial condition

$$T(r, 0) = T_0 \quad (2)$$

and with one and the same boundary condition for the convective heat transfer valid for the entire surface of the logs:

- during the process of logs freezing:

$$\frac{\partial T(0, \tau)}{\partial r} = - \frac{\alpha_r^{\text{fr}}(\tau)}{\lambda_{\text{sr}}(\tau)} \left[ T_{\text{sr}}^{\text{fr}}(\tau) - T_{\text{m}}^{\text{fr}}(\tau) \right], \quad (3)$$

- during the process of logs defrosting:

$$\frac{\partial T(0, \tau)}{\partial r} = \frac{\alpha_r^{\text{dfr}}(\tau)}{\lambda_{\text{sr}}(\tau)} \left[ T_{\text{sr}}^{\text{dfr}}(\tau) - T_{\text{m}}^{\text{dfr}}(\tau) \right]. \quad (4)$$

It must be noted that the model presented by equations (1) ÷ (4) can also be solved with different mathematically described boundary conditions and heat transfer coefficients within them. The difference between the thermo-physical characteristics of the logs' bark and the logs' wood has not been taken into consideration in the model.

## 2.2 Change in the temperature of the freezing and the defrosting mediums

It is possible to have two cases for freezing of different materials in freezers. The first case is when the material is put into a working freezer with a constant unchanged temperature in it and, consequently, the freezing medium temperature  $T_m(\tau) = T_{m0} = \text{const.}$

The mathematical model (1) ÷ (4) obtains more complicated boundary conditions in the second case; in this case the material is put into a freezer before it is switched on. Here, the temperature of the air environment in the freezer  $T_m^{\text{fr}}$  decreases exponentially (*Figure 1*) according to the equation

$$T_m^{\text{fr}} = T_{m1}^{\text{fr}} + (T_{m0}^{\text{fr}} - T_{m1}^{\text{fr}}) \exp\left(-\frac{\tau}{\tau_{\text{exp}}^{\text{fr}}}\right). \quad (5)$$

The defrosting of the frozen materials after the freezer's door is opened is realized at the exponential increase of the air temperature (*Figure 1*) according to the equation

$$T_m^{\text{dfr}} = T_{m1}^{\text{dfr}} - (T_{m1}^{\text{dfr}} - T_{m1}^{\text{fr}}) \exp\left(-\frac{\tau - \tau_{\text{fr}}}{\tau_{\text{exp}}^{\text{dfr}}}\right). \quad (6)$$

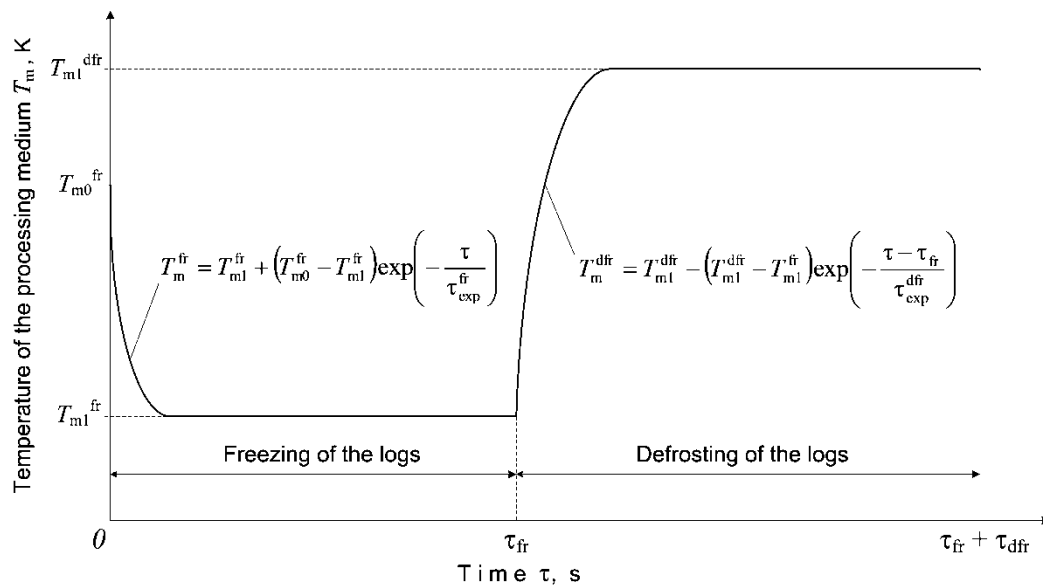


Figure 1. Exponential change in the air medium during the freezing and the defrosting of logs

## 2.3 Heat transfer coefficient between the air and the logs

It is known that the heating or cooling of logs in a gaseous (air) environment takes place through a convective heat exchange between the surfaces of the logs and the moving environment. If the movement is caused by differences in the density of the gas as a consequence of temperature differences in it, it is acceptable to call the convection free (not organized, natural).

As a rule, the freezing of wood materials at atmospheric conditions or in a freezer takes place in the conditions of free convection. For the calculation of the heat transfer coefficient in such conditions of heating or cooling of horizontally situated logs Chudinov (1968) suggests the following experimentally determined equation:

$$\alpha_r = 0.997 \sqrt[4]{\frac{\Delta T}{R}}. \quad (7)$$

It can be assumed that the heat transfer coefficients during freezing and defrosting of the materials are not equal to each other. Real free air convection is observed only during the freezing and defrosting of logs at natural atmospheric conditions or during the defrosting of frozen materials after the freezer's door is opened. That is why it can be written as:

$$\alpha_r^{\text{fr}} \approx 0.997 \sqrt[4]{\frac{\Delta T}{R}} = 0.997 \sqrt[4]{\frac{T(0, \tau) - T_m^{\text{fr}}(\tau)}{R}}, \quad (8)$$

$$\alpha_r^{\text{dfr}} = 0.997 \sqrt[4]{\frac{\Delta T}{R}} = 0.997 \sqrt[4]{\frac{T_m^{\text{dfr}}(\tau) - T(0, \tau)}{R}}. \quad (9)$$

More precise equations for the determination of the heat transfer coefficients between the surfaces of the logs in radial direction and the freezing or defrosting mediums can be obtained after suitable experiments have been carried out.

The model presented by equations (1) ÷ (4) can also be solved with different mathematically described heat transfer coefficients for separate parts of the logs' surfaces. The analysis and presentation of such cases are beyond the scope of this article.

## 2.4 Thermo-physical characteristics of the logs during their freezing and defrosting

The solution of the non-linear 1D mathematical model of the freezing and defrosting processes of logs, which is presented by the equations (1) ÷ (9), can be realized using the mathematical descriptions of the effective heat capacity of the frozen and non-frozen wood,  $c_e$ , the thermal conductivity of the wood in radial direction,  $\lambda_r$ , and the density of frozen and non-frozen wood,  $\rho$ , given in Deliiski (2011, 2013b). With the help of the mathematical description of  $\lambda_r$ , the current values of the thermal conductivity on the logs' surfaces in radial direction  $\lambda_{sr}(0, \tau)$ , which participates in eqs. (3) and (4), can also be calculated during the solution of the model.

The thermal conductivity of the wood can be calculated with the help of the following equations for  $\lambda(T, u, \rho_b, u_{\text{fsp}})$ :

$$\lambda = \lambda_{273.15} \gamma [1 + \beta(T - 273.15)], \quad (10)$$

$$\lambda_{273.15} = K_{\text{ad}} \nu [0.165 + (1.39 + 3.8u)(3.3 \cdot 10^{-7} \rho_b^2 + 1.015 \cdot 10^{-3} \rho_b)], \quad (11)$$

$$\nu = 0.15 - 0.07u \quad @ \quad u \leq u_{\text{fsp}} + 0.1 \text{ kg} \cdot \text{kg}^{-1}, \quad (12)$$

$$\nu = 0.1284 - 0.013u \quad @ \quad u > u_{\text{fsp}} + 0.1 \text{ kg} \cdot \text{kg}^{-1}. \quad (13)$$

Equations for the calculation of the variables  $\gamma$  and  $\beta$  in eq. (10) for frozen and non-frozen wood from different wood species and an algorithm for usage of the mathematical description of  $\lambda$  during the solution of the model are given in Deliiski (2013a).

The ice in the wood can be formed from the freezing of hygroscopically bound water or from both the bound and the free water in the wood. It is widely accepted that the phase transition of water into ice and vice versa can be expressed with the help of the so-called “latent heat” in the ice of the frozen body. When solving problems connected to transient heat conduction in frozen wood, it makes sense to include the latent heat in the so-called effective specific heat capacity  $c_e$  (Chudinov 1966, 1968), which is equal to the sum of the wood’s own specific heat capacity  $c$  and the specific heat capacity of the wood which is frozen only with bound water or contains both bound and free water within it. As an example, in *Figure 1*, the symbols of the thermo-physical characteristics of the frozen and non-frozen wood are present, which are necessary for the computation of the temperature distribution in the wood during heating aimed at its defrosting in the respective temperature diapasons and for the computation of the energy consumption for the heating of the wood.

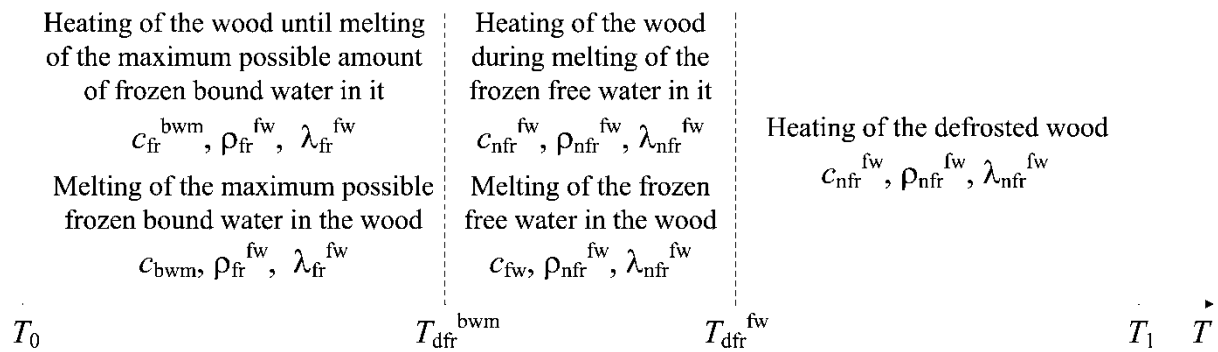


Figure 2. Using different specific heat capacities –  $c$ , thermal conductivities –  $\lambda$  and densities –  $\rho$ , for the calculation of the temperature distribution in frozen and non-frozen wood when  $u > u_{fsp}$

A mathematical description of these characteristics is given in Deliiski (2013b). When modelling processes connected with non-stationary temperature distribution in frozen wood with  $u > u_{fsp}$  during its defrosting, it is necessary to take into consideration that the effective specific heat capacity,  $c_e$ , for the separate temperature diapasons on *Figure 2* is equal to:

$$c_e = c_{fr}^{bwm} + c_{bwm} \quad @ \quad T_0 \leq T \leq T_{dfr}^{bwm}, \quad (14)$$

$$c_e = c_{nfr}^{fw} + c_{fw} \quad @ \quad T_{dfr}^{bwm} < T \leq T_{dfr}^{fw}, \quad (15)$$

$$c_e = c_{nfr}^{fw} \quad @ \quad T_{dfr}^{fw} < T \leq T_1. \quad (16)$$

During the computations of the freezing and defrosting processes of logs, effective heat capacities and densities of the wood subjected to thawing have been used and are shown in the mathematical descriptions of thermal conductivities below *Figure 2* (Deliiski 2011, 2013a, 2013b). The small difference (so named hysteresis) between these thermo-physical characteristics, and also between the temperatures  $T_{fr}^{bwm}$  and  $T_{dfr}^{bwm}$ , and between  $T_{fr}^{fw}$  and  $T_{dfr}^{fw}$  during the freezing and defrosting of the wood (Chudinov 1966, 1968) needs to be additionally studied and mathematically described.

### 3 RESULTS AND DISCUSSION

The abovementioned mathematical descriptions of  $T_m^{\text{fr}}$ ,  $T_m^{\text{dfr}}$ ,  $\alpha_r^{\text{fr}}$  and  $\alpha_r^{\text{dfr}}$  have been introduced before by the first co-author who earlier created and later modified a non-stationary model of the heating and cooling of cylindrical wood materials (Deliiski 2005, 2011, 2013b). This model is presented in common form by the eqs. (1) ÷ (4).

The updated model with the descriptions of  $T_m^{\text{fr}}$ ,  $T_m^{\text{dfr}}$ ,  $\alpha_r^{\text{fr}}$ , and  $\alpha_r^{\text{dfr}}$  has been solved with the help of explicit schemes of the finite difference method which, in a way, is analogous to the one used and described in (Deliiski 2009, 2011) for the solution of a model of the heating and cooling process of cylindrical wood materials. During the computations, these schemes allow for the determination of the temperature at each node of the calculation mesh using the current values of the thermo-physical characteristics of the frozen or non-frozen wood depending on the momentous aggregate state of the bound and free water in the wood in separate nodes.

To help with the solutions of the updated model, a software program has been prepared in the calculation environment of Visual FORTRAN Professional, which is a part of the office-package of Windows. The program helped carry out computations for the determination of the 1D change of temperature in beech logs (*Fagus Silvatica* L.) subjected to 50 hours freezing at  $t_{m1}^{\text{fr}} = -20$  °C and a following 50 h defrosting at  $t_{m2}^{\text{dfr}} = -20$  °C.

The freezing and subsequent defrosting of logs with a diameter of  $D = 0.24$  m (i.e. with a radius of  $R = 0.12$  m), initial wood temperature  $t_0 = 20$  °C and three values of the wood moisture content:  $u = 0.4$  kg · kg<sup>-1</sup>,  $u = 0.8$  kg · kg<sup>-1</sup>, and  $u = 1.2$  kg · kg<sup>-1</sup> have been studied. The moisture content of the beech logs is usually situated in this range of  $u$ , which are used for the production of veneer. All logs with such  $u$  contain a maximum possible quantity of bound water. Besides this, the logs with  $u = 0.4$  kg · kg<sup>-1</sup> contain a relatively little amount of free water, the log with  $u = 0.8$  kg · kg<sup>-1</sup> contains a significant quantity of free water and the log with  $u = 1.2$  kg · kg<sup>-1</sup> contains almost a maximum possible quantity of free water.

The decreasing of  $t_m^{\text{fr}}$  from the value of  $t_{m0}^{\text{fr}} = t_0 = 20$  °C to  $t_{m1}^{\text{fr}} = -20$  °C = const and the following increasing of  $t_m^{\text{dfr}}$  from  $t_{m1}^{\text{fr}} = -20$  °C = const to  $t_{m1}^{\text{dfr}} = 20$  °C = const go exponentially with time constants  $\tau_{\text{exp}}^{\text{fr}} = \tau_{\text{exp}}^{\text{dfr}} = 3600$  s. The calculated values according to eqs. (5) and (6) exponential change of  $t_m^{\text{fr}}$  and  $t_m^{\text{dfr}}$  can be seen in the *Figure 2* for the curve of  $t_m$ .

The duration of 50 h of the logs freezing at  $t_{m1}^{\text{fr}} = -20$  °C has been proven as being enough for the reaching of a complete freezing of the free water in the all of the studied logs. The calculations were done with average values of basic density of beech wood  $\rho_b = 560$  kg · m<sup>-3</sup> and fiber saturation point at 293.15 K (i.e. at 20 °C) of this wood  $u_{\text{fsp}}^{293.15} = 0.31$  kg · kg<sup>-1</sup> (Nikolov – Videlov 1987, Pozgaj et al. 1997). A coefficient  $K_r = K_{\text{ad}} = 1.35$  (Deliiski 2003, 2013a) in eq. (11) for the beech wood was used.

*Table 1* shows the computed distribution of the temperature in 4 equally distant from each other nodes of the calculation mesh in the central cross-section of the beech log with  $u = 0.8$  kg · kg<sup>-1</sup> at every 2 h of the freezing and of the following defrosting processes. The corresponding input data, which is used for the solution of the 1D model, is underlined in *Table 1*. The remaining input data, which is not underlined in this table, relates mainly to the

parameters of the equipment with which the thermal treatment of the wood materials with the aim of their freezing or defrosting is carried out. Using this input data, the energy parameters of the freezing or defrosting process and the efficiency from the usage of the equipment can be calculated. The fourth column from right to left on *Table 1* shows the calculated values of  $\alpha_r$  according to eq. (8) and the last two columns of *Table 1* show the calculated values of the wood thermal conductivities on the log's surface,  $\lambda_{sr}$ , and in the center of the log,  $\lambda_c$ , during the freezing and defrosting processes.

In *Figure 3* the computed change in the freezing and in the defrosting medium temperatures,  $t_m^{fr}$  and  $t_m^{dfr}$  respectively (both temperatures are shown as  $t_m$  on the figure), in the surface temperature of the log,  $t_{sr}^{fr}$  and  $t_{sr}^{dfr}$  (both of them are shown as  $t_{sr}$  on the figure) and also in the temperature in the central points of the log,  $t_c$ , during the freezing and subsequent defrosting, depending on the wood moisture content,  $u$ , is shown.

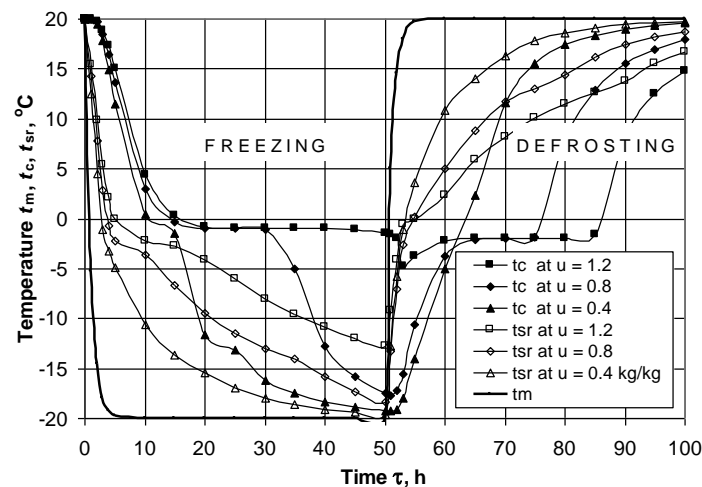


Figure 3. Change in  $t_m$ ,  $t_{sr}$ , and  $t_c$  of beech logs with  $D = 0.24$  m and  $t_0 = 20$  °C during their 50 h freezing at  $-20$  °C and following 50 h defrosting at  $20$  °C, depending on  $u$

In *Figure 4* the computed change in the heat transfer coefficient between the surfaces of the logs in radial direction,  $\alpha_r$ , and the freezing and defrosting mediums during the studied processes, depending on  $u$  is shown.

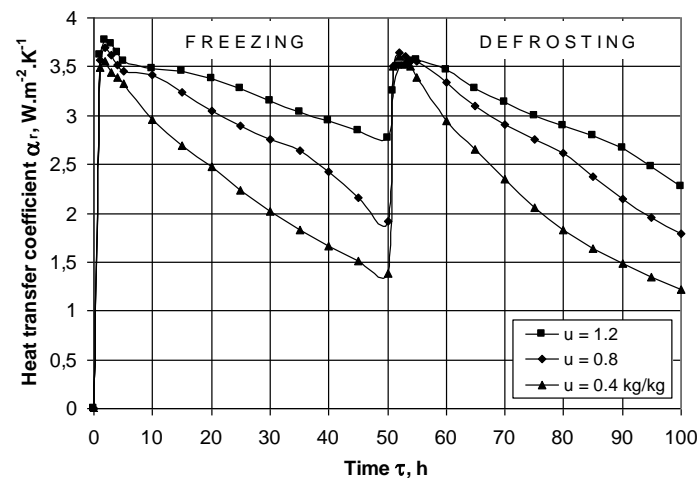


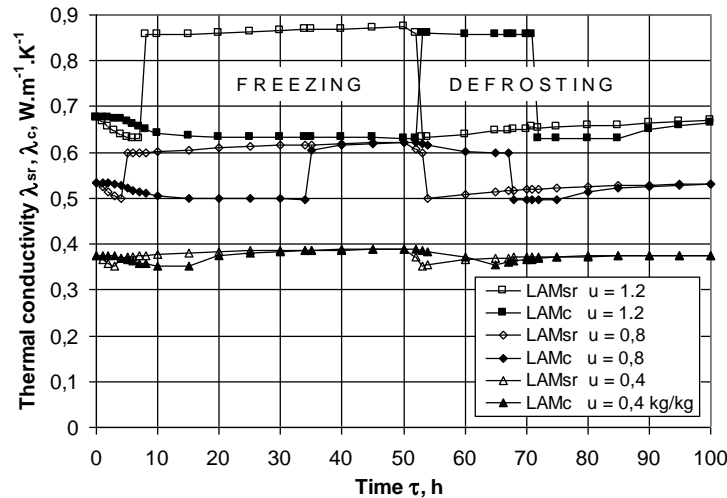
Figure 4. Change in  $\alpha_r$  of beech logs with  $D = 0.24$  m and  $t_0 = 20$  °C during their 50 h freezing at  $-20$  °C and following 50 h defrosting at  $20$  °C, depending on  $u$



Table 1. Change in  $T_m^{\text{fr}}$  and  $T_m^{\text{dfr}}$  (third column),  $T_{\text{sr}}^{\text{fr}}$  and  $T_{\text{sr}}^{\text{dfr}}$  (fourth column),  $\alpha_r$  (fourth column from right), in  $\lambda_{\text{sr}}$  and  $\lambda_c$  (second and first column from right), and in  $t$  in 4 equally distant from each other characteristic points of the central cross section of a beech log with  $D = 0.24$  m,  $t_0 = 20^\circ\text{C}$ , and  $u = 0.8 \text{ kg}\cdot\text{kg}^{-1}$  during every 2 h of its freezing at  $t_{\text{ml}}^{\text{fr}} = -20^\circ\text{C}$  and during the subsequent defrosting at  $t_{\text{ml}}^{\text{dfr}} = 20^\circ\text{C}$

I N P U T D A T A													
FREEZING AND DEFROSTING OF A BEECH LOG WITH DIAMETER OF 0,24 m													
Kq=11 M= 11 N= 0 KD= 1 Ry=560. Kwr=1.35 Kwpr=1.78 U=0.800 Ufsp293=0.31 D=2.4 L= 9.0													
to= 20.0 tmo= 20.0 tmfr=-20.0 tmdfr= 20.0 t3= 0.0 t4= 0.0 dtm=.001 t01= 0. dTAU= 80													
Tfr=3600. Tdfr=3600. T3= 0. dT3= 0. dtm3= 0. T4= 0. T5= 0. TAUproc.=360000 INT= 7200													
ds=.008 Si=.10 Roi=120. Ai=.00000022 dFa=0.05 KK=.2 tcenter=-17.45 dtwc= 0.1 Ts= 0													
Pw=.30 Vw=14.39 Va=47.95 tbi= 0. Sim=0.200 Xp=1.00 L-log=0.00 D-log=.24 dx=.01200													
R E S U L T S													
*****													
Time	Energy	Temperature	Temp.in	charact.	points	on R,	tav	dtav/	ALFAr	LAMsr	LAMc		
TAU	q	tm	tsr	30	60	90	120	Simpson	dtav/	Chud. Videl.	W/m.K	W/m.K	
h	kWh/m3	oC	oC	oC	oC	oC	oC	oC	oC/h	W/m2.K	W/m.K	W/m.K	
0	0.00	20.0	20.00	20.0	20.0	20.0	20.0	20.00	0.00	0.00	0.534	0.534	
2	3.28	-14.6	7.79	11.9	17.3	19.0	19.7	15.91	-3.18	3.69	4.36	0.514	0.533
4	8.49	-19.3	-0.76	3.0	10.4	14.1	16.4	9.37	-3.16	3.52	4.15	0.499	0.528
6	12.35	-19.9	-2.41	-0.1	4.9	8.4	10.8	4.75	-1.63	3.46	4.09	0.600	0.518
8	14.41	-20.0	-2.79	-0.9	2.2	4.4	6.1	2.14	-1.05	3.45	4.07	0.600	0.511
10	16.49	-20.0	-3.59	-1.1	0.7	2.0	3.0	0.49	-1.10	3.41	4.03	0.602	0.505
12	17.36	-20.0	-4.74	-1.8	-0.3	0.5	1.2	-0.67	-0.39	3.35	3.95	0.603	0.502
14	18.44	-20.0	-6.26	-3.8	-0.8	-0.3	0.1	-1.63	-0.37	3.26	3.85	0.606	0.501
16	19.75	-20.0	-7.14	-5.0	-1.0	-0.8	-0.6	-2.39	-0.52	3.21	3.79	0.607	0.499
18	20.14	-20.0	-8.41	-6.4	-1.1	-1.0	-0.9	-3.01	-0.19	3.13	3.69	0.609	0.499
20	21.00	-20.0	-9.50	-7.8	-1.7	-1.0	-1.0	-3.78	-0.27	3.05	3.60	0.610	0.499
22	22.27	-20.0	-10.33	-8.8	-3.7	-1.1	-1.0	-4.57	-0.37	2.99	3.53	0.612	0.499
24	23.03	-20.0	-11.14	-9.7	-4.6	-1.2	-1.0	-5.18	-0.73	2.92	3.45	0.613	0.499
26	23.49	-20.0	-11.87	-10.6	-6.2	-1.8	-1.0	-5.99	-0.28	2.86	3.38	0.614	0.499
28	24.78	-20.0	-12.44	-11.3	-7.3	-3.6	-1.0	-6.83	-0.37	2.81	3.32	0.615	0.499
30	25.12	-20.0	-13.00	-11.9	-8.1	-4.3	-1.1	-7.42	-0.24	2.76	3.25	0.616	0.499
32	26.01	-20.0	-13.47	-12.5	-9.1	-5.8	-1.2	-8.28	-0.31	2.71	3.20	0.616	0.498
34	27.30	-20.0	-13.89	-13.0	-9.8	-6.8	-1.7	-9.12	-0.48	2.66	3.15	0.617	0.498
36	28.52	-20.0	-14.32	-13.5	-11.0	-9.1	-7.5	-10.94	-1.01	2.62	3.09	0.617	0.607
38	29.46	-20.0	-15.03	-14.4	-12.7	-11.5	-10.6	-12.72	-0.79	2.53	2.99	0.619	0.612
40	30.19	-20.0	-15.82	-15.3	-14.1	-13.2	-12.7	-14.12	-0.62	2.42	2.86	0.620	0.615
42	30.76	-20.0	-16.53	-16.1	-15.2	-14.6	-14.1	-15.24	-0.49	2.31	2.73	0.621	0.617
44	31.21	-20.0	-17.12	-16.8	-16.1	-15.6	-15.3	-16.12	-0.40	2.21	2.61	0.622	0.619
46	31.56	-20.0	-17.61	-17.4	-16.8	-16.4	-16.2	-16.83	-0.32	2.11	2.49	0.622	0.620
48	31.85	-20.0	-18.01	-17.8	-17.4	-17.1	-16.9	-17.40	-0.25	2.01	2.38	0.623	0.621
50	32.08	-20.0	-18.34	-18.2	-17.8	-17.6	-17.5	-17.85	-0.20	1.92	2.27	0.623	0.622
52	30.00	14.5	-7.07	-10.3	-14.9	-16.5	-17.2	-13.80	3.22	3.65	4.32	0.607	0.622
54	25.73	19.3	-0.15	-3.1	-8.4	-11.3	-13.1	-7.79	2.05	3.56	4.20	0.500	0.616
56	23.67	19.9	0.48	-2.0	-5.0	-7.0	-8.5	-4.81	1.04	3.56	4.20	0.501	0.609
58	21.64	20.0	2.78	-1.3	-3.3	-4.6	-5.5	-2.84	0.65	3.45	4.07	0.505	0.605
60	20.23	20.0	4.96	1.6	-2.4	-3.2	-3.8	-1.36	0.55	3.34	3.94	0.509	0.602
62	18.49	20.0	6.61	3.9	-2.0	-2.4	-2.8	-0.05	0.54	3.24	3.83	0.511	0.600
64	17.27	20.0	8.01	5.7	-1.6	-2.1	-2.2	1.03	0.55	3.15	3.72	0.514	0.600
66	15.64	20.0	9.27	7.2	0.8	-2.0	-2.0	2.10	0.69	3.07	3.62	0.516	0.599
68	13.56	20.0	10.41	8.6	2.1	-1.8	-2.0	2.96	1.08	2.98	3.52	0.518	0.497
70	12.71	20.0	11.32	9.7	4.2	-1.2	-2.0	4.05	0.37	2.91	3.43	0.519	0.497
72	11.81	20.0	12.06	10.6	5.8	1.3	-2.0	5.18	0.42	2.84	3.36	0.520	0.497
74	11.03	20.0	12.71	11.4	7.0	2.9	-1.9	6.18	0.54	2.78	3.29	0.522	0.497
76	10.36	20.0	13.28	12.1	8.0	4.0	-1.6	7.03	1.61	2.73	3.22	0.522	0.498
78	8.89	20.0	13.76	12.7	9.1	6.2	3.7	8.89	1.24	2.68	3.16	0.523	0.506
80	7.21	20.0	14.38	13.5	11.0	9.2	7.9	11.01	0.91	2.61	3.08	0.524	0.514
82	5.93	20.0	15.14	14.4	12.5	11.3	10.4	12.62	0.71	2.52	2.97	0.526	0.518
84	4.92	20.0	15.86	15.3	13.8	12.9	12.2	13.88	0.56	2.42	2.85	0.527	0.521
86	4.11	20.0	16.47	16.0	14.8	14.1	13.6	14.89	0.46	2.32	2.74	0.528	0.523
88	3.46	20.0	16.99	16.6	15.7	15.1	14.6	15.71	0.37	2.23	2.64	0.529	0.525
90	2.92	20.0	17.42	17.1	16.3	15.8	15.5	16.38	0.30	2.15	2.54	0.529	0.526
92	2.48	20.0	17.78	17.5	16.9	16.5	16.2	16.93	0.25	2.07	2.44	0.530	0.527
94	2.11	20.0	18.09	17.9	17.3	17.0	16.8	17.38	0.21	1.99	2.35	0.530	0.528
96	1.81	20.0	18.34	18.2	17.7	17.5	17.3	17.76	0.17	1.92	2.27	0.531	0.529
98	1.56	20.0	18.56	18.4	18.0	17.8	17.7	18.07	0.14	1.86	2.19	0.531	0.530
100	1.35	20.0	18.74	18.6	18.3	18.1	18.0	18.34	0.12	1.79	2.12	0.532	0.530
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The computed change in the thermal conductivity of the surfaces of the logs in radial direction,  $\lambda_{sr}$ , and of the centres of the logs  $\lambda_c$ , depending on  $u$  during the freezing and defrosting processes is shown in *Figure 5*.



*Figure 5. Change in  $\lambda_{sr}$  and in  $\lambda_c$  of beech logs with  $D = 0.24$  m and  $t_0 = 20$  °C during their 50 h freezing at  $-20$  °C and following 50 h defrosting at  $20$  °C, depending on  $u$*

In Deliiski (2009) curves showing the percentage of water being in ice-phase on the diameter of beech logs subjected to defrosting with  $t_0 = -20$  °C as a function of  $u > u_{fsp}$  and the relationship  $\tau / D$  can be seen.

The obtained results lead to the following conclusions:

1. On the curves of characteristic points situated on the logs' centers in *Figure 3*, the specific almost horizontal sections of retention of the temperature  $t_c$  for a long period of time in the range from  $-1$  °C to  $-2$  °C can be seen, while in these points a complete freezing of the free water and after that a complete melting of the ice created by it occurs in the wood (Chudinov 1966, 1968).

It can be noted that such retention of the temperature on the logs' axes has been observed in wide experimental studies during the defrosting process of pine logs containing ice from the free water (Steinhagen 1986, Khattabi – Steinhagen 1992, 1993).

2. The character of the change in the heat transfer coefficient,  $\alpha_r$ , is almost identical during the studied freezing and defrosting processes of logs with given value of the wood moisture content (*Figure 4*). The reason for this is the equality of the difference  $t_0 - t_m^{fr}$  during the freezing with the difference  $t_m^{dfr} - t_m^{fr}$  during the defrosting for one and the same moments from the beginning of these processes at  $t_{m0}^{fr} = 20$  °C,  $t_{m1}^{fr} = -20$  °C,  $t_{m1}^{dfr} = 20$  °C, and  $\tau_{exp}^{fr} = \tau_{exp}^{dfr} = 3600$  s. At other values of these variables, the change in  $\alpha_r$  during the freezing and the defrosting would be different.

3. With the increase of the duration of the freezing and the defrosting processes, the coefficient  $\alpha_r$  decreases because of the decreasing of the difference  $\Delta T$  between the processing medium temperature and the surface temperature of the logs (see eqs. (8) and (9)).

4. The character of the change in the wood thermal conductivity on the logs' surfaces,  $\lambda_{sr}$ , and in the logs' centers,  $\lambda_c$ , is very complex (*Figure 5*). The current values of  $\lambda_{sr}$  and of  $\lambda_c$  depend not only on the wood moisture content and on the current temperature in the respective points on the logs' radiuses, but also on the momentous aggregate condition of the water in these points (Deliiski, 2013a). The larger values of  $\lambda_{sr}$  and  $\lambda_c$  in *Figure 5* related to

the central points or surfaces of the frozen logs, and the lower values of  $\lambda_{sr}$  and  $\lambda_c$  related to the logs' points with frozen free water in them at respective moments.

Table 1, Figure 3, and Figure 5 show that the complete freezing of the free water at the surface and at the center of the beech log with  $u = 0.8 \text{ kg} \cdot \text{kg}^{-1}$  occurs between 4 and 6 h and between 34 and 36 h respectively, when the temperature in these points becomes lower than  $-2^\circ\text{C}$ . Analogously, the melting of the frozen free water at the surface and at the center of this log starts between 52 and 54 h and between 66 and 68 h respectively. The complete melting of the frozen free water at the center of the log occurs between 76 and 78 h, when the temperature in this point becomes higher than  $-1^\circ\text{C}$ .

## 4 CONCLUSIONS

The present paper describes the 1D mathematical model for the computation of the temperature on the surfaces of logs suggested by the authors,  $t_{sr}$ , and the non-stationary temperature distribution along the radiuses of logs subjected to freezing and to subsequent defrosting at convective exponentially changing boundary conditions. As a base model, the heating and cooling processes of logs is used, which was created and modified earlier by the first co-author. The mechanism of the heat distribution along the radiuses of the logs during their freezing and subsequent defrosting is described by the 1D partial differential equation of heat conduction. For the solution of the model, an explicit form of the finite-difference method is used, which allows for the exclusion of any simplifications in the model.

For the numerical solution of the model, a software program has been prepared in FORTRAN, which has been input in the Visual Fortran Professional calculation environment developed by Microsoft. With the help of the program an example computation has been carried out for the determination of the 1D change in the temperature along the radiuses of beech logs with diameter 0.24 m, initial temperature  $20^\circ\text{C}$  and moisture content  $0.4 \text{ kg} \cdot \text{kg}^{-1}$ ,  $0.8 \text{ kg} \cdot \text{kg}^{-1}$  and  $1.2 \text{ kg} \cdot \text{kg}^{-1}$ , during 50 hours freezing at exponentially decreasing air temperature until reaching of  $-20^\circ\text{C}$  and during the following 50 h defrosting at exponentially increasing air temperature until reaching of  $20^\circ\text{C}$ .

The results presented in the figures in this paper show that the procedures for calculation of the non-stationary 1D temperature change in the prepared software program function well for the mutually connected processes of the freezing and the defrosting of the logs at convective boundary conditions.

The obtained results show the complex character of the change in the temperature on the logs' surfaces and along the logs' radiuses, and also of the heat transfer coefficient between the logs' surfaces and the processing freezing or defrosting air environment. Also the change in the wood thermal conductivity on the logs' surfaces and in the separate points along the logs' radiuses, especially strong depending on the aggregate condition of the water in each point at every moment of the studied processes, has a very complex character.

The presented model, after its update with new experimentally obtained data about the heat transfer coefficient and with mathematically described hysteresis between the thermo-physical characteristics of the wood during its freezing and defrosting, can be used for a science-based determination of the duration of the freezing and defrosting processes of logs at different initial and boundary conditions.

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